Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel Level 3 GCE

Thursday 22 June 2023

Afternoon (Time: 1 hour 30 minutes)

Paper reference **9FM0/3A**

Further Mathematics

Advanced

PAPER 3A: Further Pure Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

1. (a) Use Simpson's rule with 4 intervals to find an estimate for

$$\int_0^2 e^{\sin^2 x}\, dx$$

Give your answer to 3 significant figures.

(4)

Given that $\int_0^2 e^{\sin^2 x}\, dx = 3.855$ to 4 significant figures,

(b) comment on the accuracy of your answer to part (a).

(1)

(a) $\int_a^b f(x)\,dx = \frac{1}{3}h\left[y_0 + 4(y_1 + .. + y_{2n-1}) + 2(y_2 + . + y_{2n-2}) + 2n\right]$

Interval step length $= \frac{2-0}{4} = 0.5$ ①

x	0	0.5	1	1.5	2
y	1	1.258.	2.030..	2.704	2.286..

①

$e^{\sin^2 x}$

$\int_0^2 e^{\sin^2 x}\,dx \approx \frac{0.5}{3}\left[1 + 4(1.258. + 2.704) + 2(2.030.) + 2.286...\right]$

$\int_0^2 e^{\sin^2 x}\,dx \approx \frac{0.5}{3} \times 23.198..$ ①

$\int_0^2 e^{\sin^2 x}\,dx \approx 3.87$ (3 s f) ①

(b) It is accurate to 2 significant figures.

Question 1 continued

(Total for Question 1 is 5 marks)

2. The vertical height, h m, above horizontal ground, of a passenger on a fairground ride, t seconds after the ride starts, where $t \leqslant 5$, is modelled by the differential equation

$$t^2\frac{d^2h}{dt^2} - 2t\frac{dh}{dt} + 2h = t^3 \qquad \text{(I)}$$

(a) Given that $t = e^x$, show that

(i) $t\dfrac{dh}{dt} = \dfrac{dh}{dx}$

(ii) $t^2\dfrac{d^2h}{dt^2} = \dfrac{d^2h}{dx^2} - \dfrac{dh}{dx}$

(4)

(b) Hence show that the transformation $t = e^x$ transforms equation (I) into the equation

$$\frac{d^2h}{dx^2} - 3\frac{dh}{dx} + 2h = e^{3x}$$

(1)

(c) Hence show that

$$h = At + Bt^2 + \frac{1}{2}t^3$$

where A and B are constants.

(6)

Given that when $t = 1$, $h = 2.5$ and when $t = 2$, $\dfrac{dh}{dt} = -1$

(d) determine the height of the passenger above the ground 5 seconds after the start of the ride.

(5)

(a)(i) $t = e^x \implies \dfrac{dt}{dh} = e^x \times \dfrac{dx}{dh}$

Using chain rule. $t = e^u$ $\quad u = x \rightarrow \dfrac{dt}{dh} = e^x \dfrac{dh}{dx}$

$t' = e^u \quad u' = \dfrac{dx}{dh}$

$\dfrac{1}{\frac{dt}{dh}} = \dfrac{1}{e^x} \times \dfrac{1}{\frac{dx}{dh}}$ (1) $\implies \dfrac{dh}{dt} = \dfrac{1}{t} \times \dfrac{dh}{dx}$ $\quad \times t$

$t\dfrac{dh}{dt} = \dfrac{dh}{dx}$ (1)

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Question 2 continued

(a)(ii) $t\dfrac{dh}{dt} = \dfrac{dh}{dx}$ Apply chain rule.

w.r.t t

$u = t \quad v = \dfrac{dh}{dt}$

$u' = 1 \quad v' = \dfrac{d^2h}{dt^2}$

$uv' + vu' = t\dfrac{d^2h}{dt^2} + \dfrac{dh}{dt}$

$\dfrac{d}{dt}\left(\dfrac{dh}{dx}\right) = \dfrac{d}{dx}\left(\dfrac{dh}{dx}\right) \times \dfrac{dx}{dt}$

$= \dfrac{d^2h}{dx^2}\dfrac{dx}{dt}$

$t\dfrac{d^2h}{dt^2} + \dfrac{dh}{dt} = \dfrac{d^2h}{dx^2}\dfrac{dx}{dt}$ 1

$t = e^x$ so $\dfrac{dt}{dx} = e^x$

$\dfrac{dx}{dt} = \dfrac{1}{e^x} = \dfrac{1}{t}$

$t\dfrac{d^2h}{dt^2} + \dfrac{dh}{dt} = \dfrac{d^2h}{dx^2} \times \dfrac{1}{t}$

$t\dfrac{d^2h}{dt^2} = \dfrac{1}{t}\dfrac{d^2h}{dx^2} - \dfrac{dh}{dt}$

$t^2\dfrac{d^2h}{dt^2} = \dfrac{d^2h}{dx^2} - t\dfrac{dh}{dt}$

$t\dfrac{dh}{dt} = \dfrac{dh}{dx}$

$t^2\dfrac{d^2h}{dt^2} = \dfrac{d^2h}{dx^2} - \dfrac{dh}{dx}$ 1

(b) $t^2\dfrac{d^2h}{dt^2} - 2t\dfrac{dh}{dt} + 2h = t^3$

from (a)(ii)

$\dfrac{d^2h}{dx^2} - \dfrac{dh}{dx} - 2\dfrac{dh}{dx} + 2h = e^{3x} \leftarrow t = e^x$

$t\dfrac{dt}{dx} = \dfrac{dh}{dx}$ from (a)(i)

$\dfrac{d^2h}{dx^2} - 3\dfrac{dh}{dx} + 2h = e^{3x}$ 1

Question 2 continued

(c) Auxiliary Equation : $m^2 - 3m + 2 = 0$

$(m-2)(m-1) = 0$

$\therefore$ m has 2 real roots $m = 1, 2$ ①

General solution : $Ae^{x} + Be^{2x} = h$ ①

Particular Integral : $f(x) = e^{3x}$

P.I has form ke^{3x} ①

$\frac{dh}{dx} = 3ke^{3x}$ and $\frac{d^2h}{dx^2} = 9ke^{3x}$

Sub. into original equation

$9ke^{3x} - 3(3ke^{3x}) + 2(ke^{3x}) = e^{3x}$ ①

$9k - 9k + 2k = 1$

$2k = 1$

$k = \frac{1}{2}$ ①

$h = GS + PI$

$\therefore h = Ae^{x} + Be^{2x} + \frac{1}{2}e^{3x}$

$\therefore h = At + bt^2 + \frac{1}{2}t^3$ ①

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Question 2 continued

(d) When $t = 1$, $h = 2.5$:

$$2.5 = A + B + \frac{1}{2}$$

$$2 = A + B \quad 1$$

$$\frac{dh}{dt} = A + 2Bt + \frac{3}{2}t^2$$

When $t = 2$, $\frac{dh}{dt} = -1$:

$$-1 = A + 4B + 6 \quad 1$$

$$2 = A + B \qquad A = 2 - B$$

$$-7 = A + 4B \qquad -7 = 2 - B + 4B$$

$$-9 = 3B$$

$$B = -3$$

$$A = 2 - -3 = 5$$

$$\therefore h = 5t - 3t^2 + \frac{1}{2}t^3 \quad 1$$

When $t = 5$:

$$h = 5(5) - 3(5)^2 + \frac{1}{2}(5)^3 \quad 1$$

$$h = 12.5 \text{ m} \quad 1$$

(Total for Question 2 is 16 marks)

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Turn over

3. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

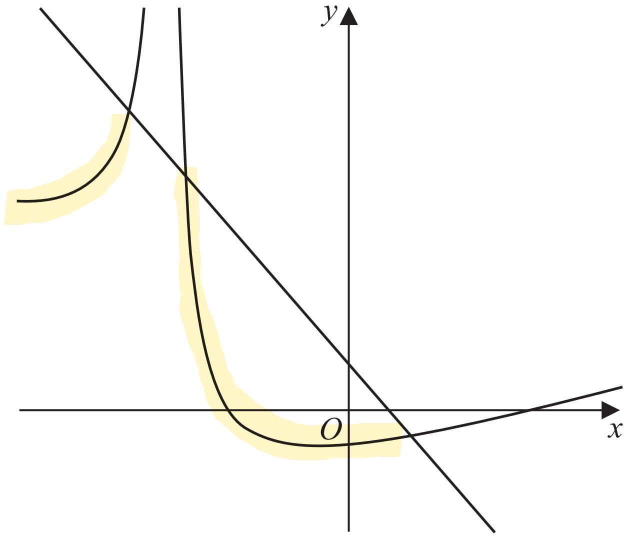

Figure 1

Figure 1 shows a sketch of the curve with equation $y = \dfrac{x^2 - 2x - 24}{|x+6|}$ and the line with equation $y = 5 - 4x$

Use algebra to determine the values of x for which

$$\frac{x^2 - 2x - 24}{|x+6|} < 5 - 4x$$

(7)

$$\frac{x^2 - 2x - 24}{|x+6|} < 5 - 4x$$

$$x^2 - 2x - 24 < |x+6|(5-4x)$$

Consider $(x+6)$ is positive.

$$x^2 - 2x - 24 < (x+6)(5-4x) \quad 1$$

$$x^2 - 2x - 24 < -4x^2 - 19x + 30$$

$$5x^2 + 17x - 54 < 0$$

$$(5x+27)(x-2) < 0 \Rightarrow x = 2, -\frac{27}{5} \quad 1$$

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Question 3 continued

Consider $(x+6)$ is negative.

$$x^2 - 2x - 24 < -(x+6)(5-4x) \quad ①$$

$$x^2 - 2x - 24 < 4x^2 + 19x - 30$$

$$3x^2 + 21x - 6 > 0$$

$$\frac{-21 \pm \sqrt{(21)^2 - 4 \times 3 \times (-6)}}{2 \times 3} \Rightarrow x = \frac{-7 \pm \sqrt{57}}{2} \quad ①$$

Just $x = \dfrac{-7-\sqrt{57}}{2}$ because $x = \dfrac{-7+\sqrt{57}}{2}$ doesn't satisfy the original inequality.

From $x = 2$, $x = -\dfrac{27}{5}$, $x = \dfrac{-7-\sqrt{57}}{2}$, curve is below line when: ← see highlighted graph

$$x < \frac{-7-\sqrt{57}}{2} \; ① \quad \text{and} \quad -\frac{27}{5} < x < 2 \; ① \quad ①$$

Turn over

Question 3 continued

Question 3 continued

(Total for Question 3 is 7 marks)

4. The ellipse E has equation

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

(a) Determine the exact value of the eccentricity of E

(2)

The points $P(4\cos\theta, 3\sin\theta)$ and $Q(4\cos\theta, -3\sin\theta)$ lie on E where $0 < \theta < \frac{\pi}{2}$
The line l_1 is the normal to E at the point P

(b) Use calculus to show that l_1 has equation

$$4x\sin\theta - 3y\cos\theta = 7\sin\theta\cos\theta$$

(4)

The line l_2 passes through the origin and the point Q
The lines l_1 and l_2 intersect at the point R

(c) Determine, in simplest form, the coordinates of R

(4)

(d) Hence show that, as θ varies, R lies on an ellipse which has the same eccentricity as ellipse E

(2)

(a) $\frac{x^2}{16} + \frac{y^2}{9} = 1$

ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ has eccentricity $b^2 = a^2(1-e^2)$

$b^2 = a^2(1-e^2)$

$9 = 16(1-e^2)$ 1

$-7 = -16e^2$

$\frac{7}{16} = e^2 \implies e = \frac{\sqrt{7}}{4}$ 1

(b) $\frac{x^2}{16} + \frac{y^2}{9} = 1$

differentiate w.r.t x

$\frac{x}{8} + \frac{2y\frac{dy}{dx}}{9} = 0$ 1

Question 4 continued

$$\frac{2y\frac{dy}{dx}}{9} = -\frac{x}{8}$$

$\times 9$

$$2y\frac{dy}{dx} = -\frac{9x}{8}$$

$\div 2y$

$$\frac{dy}{dx} = -\frac{9x}{16y}$$

At point $P(4\cos\theta, 3\sin\theta)$:

$$\frac{dy}{dx} = -\frac{9 \times 4\cos\theta}{16 \times 3\sin\theta} = -\frac{3\cos\theta}{4\sin\theta} \quad (1)$$

$$m_N = \frac{4\sin\theta}{3\cos\theta} \leftarrow m \times m_N = -1$$

$$y - 3\sin\theta = \frac{4\sin\theta}{3\cos\theta}(x - 4\cos\theta) \quad (1)$$

$$3y\cos\theta - 9\sin\theta\cos\theta = 4x\sin\theta - 16\sin\theta\cos\theta$$

$$4x\sin\theta - 3y\cos\theta = 7\sin\theta\cos\theta \quad (1)$$

(c) $Q(4\cos\theta, -3\sin\theta)$ so gradient of $l_2 = -\dfrac{3\sin\theta}{4\cos\theta}$

$l_2 : \; y = -\dfrac{3\sin\theta}{4\cos\theta}x \quad (1)$

$$4x\sin\theta - 3x\left(-\frac{3\sin\theta}{4\cos\theta}\right)\cos\theta = 7\sin\theta\cos\theta$$

substitute l_2 into (b)

Question 4 continued

$$4x\sin\theta + \frac{9x\sin\theta\cos\theta}{4\cos\theta} = 7\sin\theta\cos\theta \quad ①$$

$$4x\sin\theta + \frac{9}{4}x\sin\theta = 7\sin\theta\cos\theta$$

$$4x + \frac{9}{4}x = 7\cos\theta$$

$$\frac{25}{4}x = 7\cos\theta$$

$$x = \frac{28\cos\theta}{25} \quad ①$$

$$L_2: \quad y = -\frac{3\sin\theta}{4\cos\theta} \times \frac{28\cos\theta}{25}$$

$$y = -\frac{84\sin\theta\cos\theta}{100\cos\theta}$$

$$y = -\frac{21\sin\theta}{25} \quad ①$$

(d) $x = \dfrac{28\cos\theta}{25}$ and $y = -\dfrac{21\sin\theta}{25}$

$\therefore$ $a = \dfrac{28}{25}$ and $b = -\dfrac{21}{25}$

ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ has parametric form $(a\cos\theta, b\sin\theta)$

$$\left(\frac{21}{25}\right)^2 = \left(-\frac{28}{25}\right)^2(1-e^2)$$

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Question 4 continued

$$\frac{441}{625} = \frac{784}{625}(1-e^2) \quad 1$$

$$\frac{-343}{625} = -e^2$$

$$\therefore \quad e = \frac{\sqrt{7}}{4}$$

Of form $(a\cos\phi, b\sin\phi)$ where $\phi = -\theta$ so an ellipse, and $e = \frac{\sqrt{7}}{4}$ as required. 1

(Total for Question 4 is 12 marks)

5. (a) Show that the substitution $t = \tan\left(\frac{x}{2}\right)$ transforms the integral

$$\int \frac{1}{2\sin x - \cos x + 5}\,dx$$

into the integral

$$\int \frac{1}{3t^2 + 2t + 2}\,dt$$

(4)

(b) Hence determine

$$\int \frac{1}{2\sin x - \cos x + 5}\,dx$$

(4)

(a) $t = \tan\left(\frac{x}{2}\right)$

$\frac{dt}{dx} = \frac{1}{2}\sec^2\left(\frac{x}{2}\right)$

$\sec^2 x = 1 + \tan^2 x$

$\frac{dt}{dx} = \frac{1}{2}\left[1 + \tan^2\left(\frac{x}{2}\right)\right]$

$\frac{dt}{dx} = \frac{1}{2}\left[1 + t^2\right]$

$\frac{dt}{dx} = \frac{1 + t^2}{2}$ ① $\Rightarrow$ $dx = \frac{2dt}{1+t^2}$

Using t-formulae, $\sin x = \frac{2t}{1+t^2}$ and $\cos x = \frac{1-t^2}{1+t^2}$

$2\sin x - \cos x + 5 = 2\left(\frac{2t}{1+t^2}\right) - \frac{1-t^2}{1+t^2} + 5$ ①

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Question 5 continued

$$\int \frac{1}{2\left(\frac{2t}{1+t^2}\right) - \frac{1-t^2}{1+t^2} + 5} \times \frac{2dt}{1+t^2} \quad ①$$

$$= \int \frac{2}{4t - 1 + t^2 + 5(1+t^2)} dt$$

$$= \int \frac{1}{3t^2 + 2t + 2} dt \quad ①$$

(b) $$\frac{1}{3}\int \frac{1}{t^2 + \frac{2}{3}t + \frac{2}{3}} dt = \frac{1}{3}\int \frac{1}{(t+\frac{1}{3})^2 + \frac{2}{3} - \frac{1}{9}} dt \quad ①$$

$$\int \frac{1}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$$

$$= \frac{1}{3}\int \frac{1}{(t+\frac{1}{3})^2 + \frac{5}{9}} dt \quad ①$$

$$= \frac{1}{3} \times \frac{1}{\sqrt{\frac{5}{9}}} \tan^{-1}\left(\frac{t+\frac{1}{3}}{\sqrt{\frac{5}{9}}}\right) + c \quad ①$$

$$= \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{3t+1}{\sqrt{5}}\right)$$

$$= \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{3\tan\left(\frac{x}{2}\right)+1}{\sqrt{5}}\right) \quad ①$$

Question 5 continued

Question 5 continued

(Total for Question 5 is 8 marks)

6. $$y = \ln(e^{2x}\cos 3x) \qquad -\frac{1}{2} < x < \frac{1}{2}$$

(a) Show that

$$\frac{dy}{dx} = 2 - 3\tan 3x$$

(2)

(b) Determine $\frac{d^4y}{dx^4}$

(3)

(c) Hence determine the first 3 non-zero terms in ascending powers of x of the Maclaurin series expansion of $\ln(e^{2x}\cos 3x)$, giving each coefficient in simplest form.

(3)

(d) Use the Maclaurin series expansion for $\ln(1 + x)$ to write down the first 4 non-zero terms in ascending powers of x of the Maclaurin series expansion of $\ln(1 + kx)$, where k is a constant.

(1)

(e) Hence determine the value of k for which

$$\lim_{x\to 0}\left(\frac{1}{x^2}\ln\frac{e^{2x}\cos 3x}{1+kx}\right)$$

exists.

(3)

(a) $y = \ln(e^{2x}\cos 3x)$

$y = \ln(e^{2x}) + \ln(\cos 3x)$

$\frac{d}{dx}\ln(f(x)) = \frac{1}{f(x)} \times f'(x)$

$y = 2x + \ln(\cos 3x)$

$\frac{dy}{dx} = 2 + -3\sin 3x \times \frac{1}{\cos 3x}$ ①

$\frac{dy}{dx} = 2 - \frac{3\sin 3x}{\cos 3x}$

$\frac{dy}{dx} = 2 - 3\tan 3x$ ①

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Question 6 continued

(b) $\frac{d^2y}{dx^2} = -9\sec^2 3x$ ①

use chain rule:

$\frac{d}{dx}\sec^2 x = \sec^2 x \tan x$

$\frac{d^3y}{dx^3} = -54\sec^2 3x\tan 3x$ ①

$u = -54\sec^2 3x \qquad v = \tan 3x$

$u' = -324\sec^2 3x\tan 3x \qquad v' = 3\sec^2 3x$

$uv' + vu' = -162\sec^4 3x + 324\sec^2 3x\tan^2 3x$

$\frac{d^4y}{dx^4} = -324\sec^2 3x\tan^2 3x - 162\sec^4 3x$ ①

(c) $y = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \ldots$

$f(0) = 0 \leftarrow \ln(e^{2\times 0}\cos(3\times 0)) = \ln(1) = 0$

$f'(0) = 2 \leftarrow 2 - 3\tan(3\times 0) = 2 - 0$

$f''(0) = -9 \leftarrow \frac{1}{\cos(0)} = 1 \Rightarrow -9\sec^2(0) = -9\times 1$

$f'''(0) = 0$

$f''''(0) = -162$ ①

$y = 0 + 2x - 9\times\frac{x^2}{2} + 0\times\frac{x^3}{6} - 162\times\frac{x^3}{24} + \ldots$ ①

$y = 2x - \frac{9}{2}x^2 - \frac{27}{4}x^4$ ①

Question 6 continued

(d) $\ln(1+kx) = kx - \frac{k^2x^2}{2} + \frac{k^3x^3}{3} - \frac{k^4x^4}{4}$ ①

in formula book

(e) $\frac{1}{x^2}\ln\left(\frac{e^{2x}\cos 3x}{1+kx}\right) = \ln(e^{2x}\cos 3x) - \ln(1+kx)$

①

$= \frac{1}{x^2}\left[\left[2x - \frac{9}{2}x^2 - \frac{27}{4}x^4\right] - \left[kx - \frac{k^2x^2}{2} + \frac{k^3x^3}{3} - \frac{k^4x^4}{4}\right]\right]$

$= \frac{1}{x^2}\left[(2-k)x - \frac{(9-k^2)}{2}x^2 - \frac{k^3x^3}{3} - \frac{(27-k^4)}{4}x^4\right.$ ①

For the limit to exist, $2-k=0 \Rightarrow k=2$. ①

Question 6 continued

(Total for Question 6 is 12 marks)

7. With respect to a fixed origin O the point A has coordinates (3, 6, 5) and the line l has equation

$$\left(\mathbf{r} - (12\mathbf{i} + 30\mathbf{j} + 39\mathbf{k})\right) \times (7\mathbf{i} + 13\mathbf{j} + 24\mathbf{k}) = \mathbf{0}$$

The points B and C lie on l such that $AB = AC = 15$

Given that A does not lie on l and that the x coordinate of B is negative,

(a) determine the coordinates of B and the coordinates of C

(4)

(b) Hence determine a Cartesian equation of the plane containing the points A, B and C

(3)

The point D has coordinates $(-2, 1, \alpha)$, where α is a constant.

Given that the volume of the tetrahedron $ABCD$ is 147

(c) determine the possible values of α

(4)

Given that $\alpha > 0$

(d) determine the shortest distance between the line l and the line passing through the points A and D, giving your answer to 2 significant figures.

(4)

(a) $(r - a) \times b = 0$ where a is a point on the line, b is a vector parallel to the line.

$\overrightarrow{AL}$ = point on l − A

$$= \pm\left(\begin{pmatrix}12\\30\\39\end{pmatrix} + \lambda\begin{pmatrix}7\\13\\24\end{pmatrix} - \begin{pmatrix}3\\6\\5\end{pmatrix}\right) \quad ①$$

$$= \begin{pmatrix}9+7\lambda\\24+13\lambda\\34+24\lambda\end{pmatrix}$$

$AB = AC = 15$ so $|\overrightarrow{AL}| = 15$.

$$\left|\begin{matrix}9+7\lambda\\24+13\lambda\\34+24\lambda\end{matrix}\right| = 15$$

Question 7 continued

$(9 + 7\lambda)^2 + (24 + 13\lambda)^2 + (34 + 24\lambda)^2 = 15^2$ ①

$81 + 126\lambda + 49\lambda^2 + 576 + 624\lambda + 169\lambda^2$
$+ 1156 + 1632\lambda + 576\lambda^2 = 15^2$

$794\lambda^2 + 2382\lambda + 1588 = 0$

$\lambda^2 + 3\lambda + 2 = 0$

$(\lambda + 2)(\lambda + 1) = 0$

$\lambda = -2, \lambda = -1$ ①

When $\lambda = -2$. ← using $r = a + \lambda b$

$$\begin{pmatrix} 12 + 7(-2) \\ 30 + 13(-2) \\ 39 + 24(-2) \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \\ -9 \end{pmatrix}$$

When $\lambda = -1$.

$$\begin{pmatrix} 12 + 7(-1) \\ 30 + 13(-1) \\ 39 + 24(-1) \end{pmatrix} = \begin{pmatrix} 5 \\ 17 \\ 15 \end{pmatrix}$$

x-value of B is negative

$\therefore B = (-2, 4, 9)$

$C = (5, 17, 15)$ ①

Question 7 continued

(b) Vector equation of a plane : $r \cdot n = d$

normal vector $n = \overrightarrow{AB} \times \overrightarrow{BC}$

$$\overrightarrow{AB} = \begin{pmatrix} 9+7(-2) \\ 24+13(-2) \\ 34+24(-2) \end{pmatrix} = \begin{pmatrix} -5 \\ -2 \\ -14 \end{pmatrix}$$

$$\overrightarrow{AC} = \begin{pmatrix} 9+7(-1) \\ 24+13(-1) \\ 34+24(-1) \end{pmatrix} = \begin{pmatrix} 2 \\ 11 \\ 10 \end{pmatrix}$$

$$n = \begin{pmatrix} -5 \\ -2 \\ -14 \end{pmatrix} \times \begin{pmatrix} 2 \\ 11 \\ 10 \end{pmatrix} = \begin{pmatrix} -2\times 10 - -14\times 11 \\ -14\times 2 - -5\times 10 \\ -5\times 11 - -2\times 2 \end{pmatrix} \quad ①$$

$$= \begin{pmatrix} 134 \\ 22 \\ -51 \end{pmatrix}$$

$$\begin{pmatrix} 134 \\ 22 \\ -51 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 6 \\ 5 \end{pmatrix} = 134\times 3 + 22\times 6 + -51\times 5$$

$$\underset{\text{normal}}{\begin{pmatrix} 134 \\ 22 \\ -51 \end{pmatrix}} \cdot \underset{\text{point on the plane}}{\begin{pmatrix} 3 \\ 6 \\ 5 \end{pmatrix}} = 279 \; ① \quad \leftarrow d = a \cdot n$$

$\therefore$ cartesian form. $134x + 22y - 51z = 279$ ①

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Question 7 continued

(c) Vol. of tetrahedron $= \frac{1}{6}|a\cdot(b\times c)|$ where a, b, c are sides

$$\vec{DA} = \begin{pmatrix}3\\6\\5\end{pmatrix} - \begin{pmatrix}-2\\1\\\alpha\end{pmatrix} = \begin{pmatrix}5\\5\\5-\alpha\end{pmatrix} \quad 1$$

$$\underset{(b\times c)}{\begin{pmatrix}134\\22\\-51\end{pmatrix}} \cdot \underset{a}{\begin{pmatrix}5\\5\\5-\alpha\end{pmatrix}} = 134\times5 + 22\times5 + -51(5-\alpha) \quad 1$$

$$= 525 - 51\alpha$$

$$\text{Vol.} = \frac{1}{6}|525-51\alpha| = \pm 147 \quad 1$$

$$|525-51\alpha| = \pm 882$$

$$\alpha_1 = \frac{-882-525}{51} = -\frac{469}{17}$$

$$\alpha_2 = \frac{882-525}{51} = 7 \quad 1$$

(d) Vector $\vec{AC}$ is $\pm\begin{pmatrix}9\\24\\34\end{pmatrix}$ 1 ← from (a)

$$\vec{AD} = \begin{pmatrix}-2\\1\\7\end{pmatrix} - \begin{pmatrix}3\\6\\5\end{pmatrix} = \begin{pmatrix}-5\\-5\\2\end{pmatrix}$$

$$\vec{BC} = \begin{pmatrix}5\\17\\15\end{pmatrix} - \begin{pmatrix}-2\\4\\-9\end{pmatrix} = \begin{pmatrix}7\\13\\24\end{pmatrix}$$

Question 7 continued

$$\vec{AD} \times \vec{BC} = \begin{pmatrix} -5 \\ -5 \\ 2 \end{pmatrix} \times \begin{pmatrix} 7 \\ 13 \\ 24 \end{pmatrix}$$

$$= \begin{pmatrix} -5 \times 24 - 2 \times 13 \\ 2 \times 7 - -5 \times 24 \\ -5 \times 13 - -5 \times 7 \end{pmatrix}$$

$$= \begin{pmatrix} -146 \\ 134 \\ -30 \end{pmatrix} \quad 1$$

$$d = \frac{|(\vec{AD} \times \vec{BC}) \cdot \vec{AL}|}{\sqrt{146^2 + 134^2 + 30^2}}$$

$$d = \frac{\left| \begin{pmatrix} -146 \\ 134 \\ -30 \end{pmatrix} \cdot \begin{pmatrix} 9 \\ 24 \\ 34 \end{pmatrix} \right|}{22\sqrt{83}} \quad 1$$

$$d = \frac{-146 \times 9 + 134 \times 24 - 30 \times 34}{22\sqrt{83}}$$

$$d = 4.4 \quad (2 \text{ s.f}) \quad 1$$

(Total for Question 7 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS